A model of kinesthetically and visually controlled arm movement

WILLIAM. T. POWERS

The Control Systems Group, 73 Ridge Pl. Cr 510, Durango, CO 81301-8136, USA.
email: powers_w@frontier.net

This paper describes a preliminary simulation of kinesthetic control systems that operate a humanoid arm having three degrees of freedom. The design is in part a literal interpretation of the stretch and tendon reflexes considered as control systems. A second level of control converts independent control of three joint angles into a trio of systems controlling the tip of the arm in pitch, yaw and distance coordinates centered on the shoulder. The basic properties of muscles are included, and the arm movements are calculated using equations describing the physical dynamics of the arm. A "visual servo" level of control is included in preliminary form. The model exhibits realistic behavior, producing stable and fast control without computing either inverse kinematics or inverse dynamics.   © 1999 Academic Press

1. Introduction

Models of human arm position control (and other simple behaviors) have been of interest for many years. During the 1950s and early 1960s, it was assumed that such models would naturally take the form of negative feedback control systems. Realistic analogue-computer simulations, however, could not be done at reasonable cost with the computing equipment then available in most laboratories; proposed models of human behavior were primarily hypothetical. The simulation of nonlinear differential equations of physical systems in the environment was particularly difficult. Only a handful of behavioral scientists ever learned and simulated the properties of negative feedback control systems before that whole approach was overshadowed by the concept of the brain as a digital computer.

The digital frame of mind led back to, or helped to preserve, a view of the brain that was put forth by the neurologist Sherrington at the beginning of the 20th century. In this view, higher "executive" centers in the brain issued general commands which were sent to lower centers where they were elaborated into more specific commands. Ultimately, all the proliferated detailed commands were funneled into the "final common pathway" where they commanded muscle contractions and subsequent physical events which we recognize as overt behavior. Even after the view of the brain as a strictly digital computer was largely abandoned, this command-driven concept persisted.

Neither Sherrington nor those who adopted his theory recognized the problems that this view would present to a modeler. There is no unique path from the general to the

particular. An executive decision to "Buy 100 shares of AT&T" does not take into account that the stock market is closed today, that one's broker is sick at home, that one has forgotten his telephone number anyway, and that one has laryngitis. Unpacked, this decision is equivalent to saying "Generate any specific commands for action required in order that the consequences can be correctly described as my having bought 100 shares of AT&T". There is a certain flavor of " … and then a miracle occurs" in this concept.

The specific commands required to produce a predefined result depend on whatever processes may intervene between the command and the final effect it is to have. So given the desired final effect, it is necessary to find the inverse of the intervening processes; that is what determines the command necessary to create a given result. This consequence of adopting a command-driven model has forced itself on modelers of behavior (and on roboticists) over the last several decades.

The literature of motor control has contained many examples of models in which commands for motor acts are formulated by computational processes that derive the inverse kinematics and inverse dynamics of limb movement. These models assume that goals are given in the form of desired trajectories of movement of a limb in space, with the end-points and the derivatives being specified in advance. The required end-points and derivatives are converted by inverse kinematic calculations into equivalent joint-angle derivatives and end-points; then the inverse dynamical calculations are applied to produce the torques required to cause the desired patterns of joint movement and final angles. The computing capabilities required of the brain in order to carry off this kind of feat, as well as the demands for knowledge about the physical properties of the muscles, body and external world, would be enough to give pause to anyone who did not have faith in the unlimited computing power of the brain.

This paper resurrects the analog approach to modeling behavior. The model described here uses few and simple computations. Inverse calculations are replaced by feedback processes in which no great accuracy is required. Behavior is produced not by specifying detailed trajectories of movement, entailing specifications of derivatives that must be consistent with each other as well as with the desired result, but by the generation of variable reference signals that specify positions. Instead of one large complex computation, the model uses a hierarchy of control in which no one control system has to do anything very complex.

The model presented here is not intended as a research tool; it contains some approximations and simplifications that should be treated more carefully before any conclusions about real behavior are drawn from it. The visual part of the model, in particular, is limited and is discussed only briefly. The main purpose of this model (and this paper) is to show how a hierarchical architecture, using closed loops, can explain motor behavior in a way that is far simpler than that of other approaches.

The model presented here is novel only in that the overall control problem is broken down into many local control problems, each one far simpler than the overall task. The basic block diagram used here is patterned after one proposed by Houk and Milhorn in the 1960s. The concept of "visual servo control" is well known to roboticists (Hutchinson, Hager & Corke, 1996), although full realization of visual control in robots awaits the development of visual systems approaching the vastly superior capabilities of those in sighted organisms.

The model briefly described below runs on a 386 or 486 PC with EGA or VGA graphics. A mathematics coprocessor is required to achieve real-time operation. The runnable code and the source code are released to the public domain; they can be obtained free of charge via the internet.†

2. A model of pointing behavior

In the following, we will build up a model of pointing behavior in which a three-degree-of-freedom arm reaches out to track a target moving in three dimensions. The first level of this model consists of a two-loop arrangement, a model of the combined tendon and stretch reflexes. The second level is based on sensing joint angle; it makes kinesthetic position control more exact and also changes the coordinate system in which control takes place. The third level is a sketch of a visually based control system which uses the lower kinesthetic systems to accomplish pointing a "fingertip" (the end of the forearm) at a movable target. This part of the model is working, but needs refinement.

I should mention that the hierarchical control model itself would not be hard to extend to more complex cases, but my mathematical limitations prevented me from supplying a model of the physical arm with more degrees of freedom. One motivation in writing this paper is the hope that others with greater skills in this department will see some promise in exploring the logical extensions of this approach to still more realistic models of limb control.

2.1. THE PHYSICAL MUSCLE AND ARM

In Figure 1 a model of the muscle used in this model is shown as a contractile part in series with an elastic part. The contractile part is assumed to shorten by an amount proportional to the driving signal (labelled e) that enters it from the motor neuron. The steady-state amount of shortening is assumed independent of the force generated by the muscle. This amounts to combining a very stiff series spring representing the effect of applied forces on sarcomeres with the series spring element in connective tissues. Not shown in viscous damping in the muscle; it is, however, included in the computer simulation.

The muscles span a joint, thus creating a torque about the joint. It is assumed that the conversion from muscle force to torque is independent of joint angle, a modest untruth that can be corrected in more advanced versions of this model. All measures of muscle length, rate of shortening and force are measured in angular units: angle at the joint, rate of change of angle and torque. Wherever "position" is mentioned, angular position is meant.

For each degree of freedom, a single bidirectional muscle system is defined as if two opposing muscles were involved, the driving signals for one increasing as those for the other decrease. The average driving signal sets muscle tone. Because the spring is actually

---

†For those with internet access interested in developing this model further, the computer model can be downloaded from the author's FTP page: ftp://ftp.frontier.net/users/powers_w

The file name to look for is armv2.exe. It is a self-extracting zipped file which includes source code (C language), runnable code, and a writeup. These materials may be used with no restrictions.
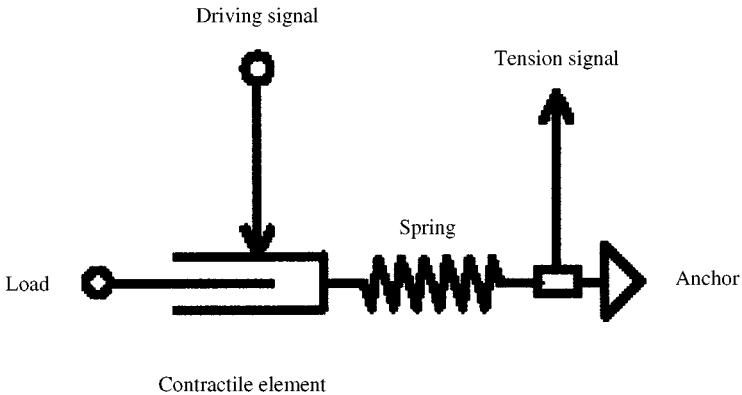
FIGURE 1. Basic muscle model. This is one-half of an opposing pair of muscles. When the driving signal entering this half increases, the signal entering the opposing half decreases (the average value sets muscle tone). When the load is held fixed relative to the anchor point (isometric contraction), a shortening of the contractile portion due to the driving signal stretches the spring component, generating a force between load and attachment. If the load is free to move, a contraction will simply move the load—when the load is stationary again the spring will be the same length as before. Real motions of the load involve a combination of these effects. The muscle also includes viscous damping, not shown here but included in the computer simulation. The tension signal represents the neural signals from tendon receptors, measuring the force produced when the load resists a contraction.

a square-law device in real muscle, setting the muscle tone for an opposed pair of springs determines the spring constant of an equivalent linear spring. When the equations for two one-way muscle systems are combined, all the resting lengths drop out and we are left with a single zero-centered linear system valid in the regions where both muscles are active; the effect of muscle tone is absorbed into the effective linear spring constant term. Also, this way of combining opposed systems automatically takes care of reciprocal connections between the reflex signals from opposing muscles, and crossovers of the driving signals as well.

The arm motions are driven by torques applied in the three degrees of freedom: pitch and yaw at the shoulder, and pitch only at the elbow. The forward dynamical equations that convert torques into angular acceleration, velocity and position were derived by Gregory Williams from inverse equations found in several robotics texts (Rivin, 1988; McCloy & Harris, 1988).

2.2. THE FIRST LEVEL OF KINESTHETIC CONTROL

2.2.1. The inner loop

The first level consists of two loops. First, consider just the tendon reflex, shown in Figure 2.

The tension generated by the muscle depends both on the driving signal E and on departures of the length X of the whole muscle from its resting length. The resting length of the contractile part is considered zero in the bidirectional model, and the equivalent spring, with a resting length of zero, resists deflections in both directions. In Figure 2 the load at position X moves to the right as the muscle contracts. The amount of shortening
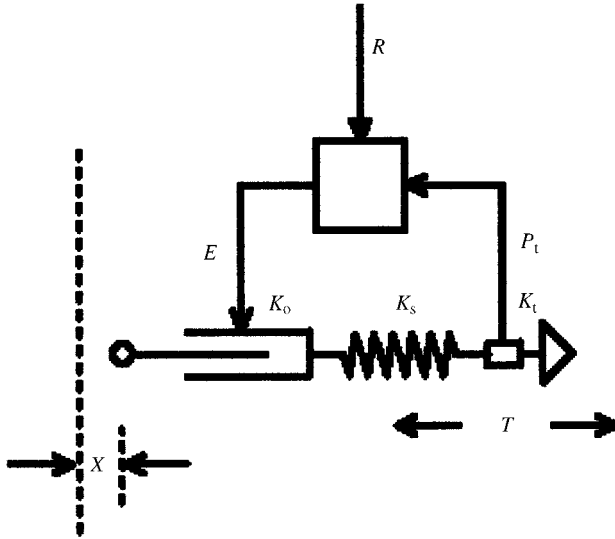
FIGURE 2. The inner control loop, or tendon reflex. See text.

is given by the driving signal E times an output conversion constant K_o, or K_oE. With the load fixed, this shortening will stretch the spring element, creating a force K_sK_oE on the load toward the right, where K_s is the spring constant in units of force per unit of stretch. The reverse of these effects should be considered as taking place simultaneously in the opposed muscle system.

If the degree of contraction is zero, then moving the load to the right by a distance X will compress the spring shown in Figure 1 (actually it will stretch the opposing spring), generating a force to the left, the negative direction, that is also proportional to the spring constant. Thus, the total tension T depends on both the amount of contraction and the position of the load relative to the neutral position:

$$T = K_s(K_oE - X). \tag{1}$$

The perceptual signal representing muscle tension P_t, is some constant K_t times the tension:

$$P_t = K_tT, \tag{2}$$

where K_t is given in neural units per unit of tension.

This signal feeds back with a negative sign to the spinal motor neuron, which also receives two positive or excitatory signals, E_g and R_a, to be discussed later, their sum being represented in Figure 2 by R. The output of the neuron (actually a balanced pair of neurons), the error signal E, is thus

$$E = R - P_t \tag{3}$$

Substituting Equations 2 and 3 into Equation 1 we arrive at the equation for the inner control loop, (Figure 2), expressing tension in terms of the net reference signal R and the

displacement X of the load:

$$T = K_s [K_o(R - K_t T) - X],$$

which reduces to

$$T = \frac{K_s K_o K_t}{1 + K_s K_o K_t} (R/K_t - X/(K_o K_t)). \tag{4}$$

The product $K_o K_s K_t$ is the "loop gain" of the inner loop: the negative of the product of all constants encountered in one trip all the way around the loop (taking the subtraction at the spinal neuron into account as a factor of -1). If this gain is much greater than 1, the expression $K_s K_o K_t /(1 + K_s K_o K_t)$ will approach unity. Designating the actual value as U, we have

$$T = U[R/K_t - X/(K_o K_t)]. \tag{5}$$

With the reference signal R held constant, a tension proportional to deflection X of the load will appear, of magnitude $X/(K_o K_t)$. This is an apparent spring constant, but note that the actual spring constant of the muscle, K_s, does not appear in this expression. In fact, the constant K_s appears only in U, in both the numerator and the denominator. The greater the loop gain, the less effect the actual muscle spring constant has on the apparent spring constant. This result is significant in the evaluation of other models, particularly the so-called mass-spring model.

If the load position is fixed at zero, and if we temporarily designate the magnitude of the combined signals E_g and R_a as an equivalent reference signal R, we have, substituting from Equation 2.

$$P_t = UR.$$

This gives us a basic way of characterizing what the inner loop, the "tendon reflex", does. It makes the sensed magnitude of tension in a tendon, P_t, match the total reference signal received by the motor neuron. The reference signal, simply by having a particular magnitude, "specifies" the magnitude at which the perceptual signal P_t is to be maintained, and thus the magnitude of tension T (equal to P_t/K_t) that is to be maintained. The degree of match depends on the loop gain, and hence the nearness of the value of U to 1. If the loop gain is 100, U will be 100/101, and the match will be accurate to 1%. This inner loop can be characterized as a force control system, or with conversion to appropriate angular units, a torque control system. We will speak of T as torque from here on.

The greater the sensitivity of the contractile part of the muscle to driving signals and the tendon sensor to tensions—that is, the larger the value of $K_o K_t$—the more independent this tension control will be of deflections of the load. Another way to say this is that as $K_o K_t$ increases, the compliance of the muscle to load forces will also increase. If the reference torque is set to zero, applying a force to the load will cause the (pair of) muscles to respond in a way that keeps the total sensed torque nearly at zero: that is the load will be moved by the applied force, and the control system will adjust the degree of muscle contraction required to maintain zero stretch in the series spring. Resistance to movements of the load will approach zero. For nonzero settings of the reference signal, a constant torque will be maintained, but it will not be affected by movements of the load. This system thus approaches pure control of the angular acceleration of the limb.

2.2.2. The outer loop

The second loop involves the stretch reflex, with a resting state determined by the magnitude of a "gamma efferent" signal, which acts as a reference signal. The gamma reference signal R_g adjusts the reference length by contracting small muscles at the poles of the spindle sensory organ, so an increase of R_g stretches the sensor at the equator by an amount K_1 per unit of neural signal. A movement of the load will stretch the sensor by an amount $-X$, meaning that a movement to the left will cause a positive sensor signal. The sum of these two effects, $K_1 R_g - X$, is multiplied by a sensitivity factor K_g to produce the gamma error signal E_g:

$$E_g = K_g(K_1 R_g - X).$$

The muscle spindle is a mechanical comparator with a reference length $K_1 R_g$, and the signal E_g is a length error signal. The arrangement is shown in Figure 3.

To the error signal we must add a rate component, for the response of the length sensor includes sensitivity to the first time derivative of sensed length, which is affected both by x and the contractions at the poles, $K_1 R_g$. A constant K_d can be adjusted to determine the relative amount of this component:

$$E_g = K_g (K_1 R_g - X) + K_d \mathrm{d}/\mathrm{d}t(K_g(K_1 R_g - X)). \tag{6}$$

Substituting this value of E_g into Equation 5, we arrive at the equation that expresses torque in terms of load position X and the two reference signals R_g and R_a—the gamma and alpha inputs to the spinal system:

$$T = U \left\{ \frac{K_g K_1 R_g + R_a}{K_t} + \frac{K_d \mathrm{d}/\mathrm{d}t\,(K_g K_1 R_g - K_g X)}{K_t} - \left(\frac{1}{K_o} + \frac{K_g}{K_t} \right) X \right\}.$$

The torque depends on the two reference signal inputs R_g and R_a, on the rate of change of R_g and of position X, and on deflection of the load by the amount X from its zero position. The values of the constants in the control loops determine the properties of this relationship.

2.3. A SINGLE-JOINT ANALYSIS

An important insight into the role of feedback in arm movement can be seen by considering a single degree of freedom, like a forearm being moved about the elbow joint with the upper arm fixed in space. Let the movable segment have a moment of inertia J: thus $T = J(\mathrm{d}^2 x/\mathrm{d}t^2)$. Substituting and rearranging, we find the differential equation for the motion of the arm under control of the two-loop control system:

$$\frac{K_t J}{K_g U} \mathrm{d}^2/\mathrm{d}t^2(X) + K_d K_g \,\mathrm{d}/\mathrm{d}t(X - K_1 R_g) + (1/K_o + K_g)X = K_g K_1 R_g + R_a K_g U$$

This is a second-order differential equation like that for a mass on a torsion spring, of the form $A(\mathrm{d}^2 X/\mathrm{d}t^2) + B(\mathrm{d}X/\mathrm{d}t) + CX = f(t)$. The reference signal made of R_g and R_a, if considered variable, is like the driving function $f(t)$.

The solution of this differential equation expresses the way the arm-joint will change angle as the sum of the two reference signals is varied. In the model itself, there are no
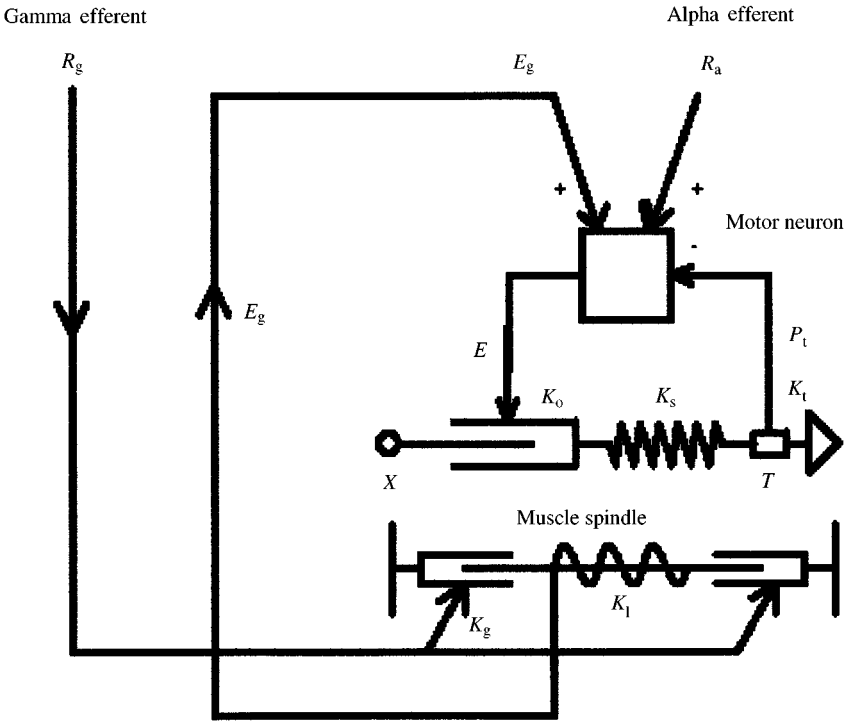
FIGURE 3. The stretch control system. Below the muscle is the muscle spindle, with small muscles at its ends and an annulospiral stretch receptor would around its middle. The end muscles are activated by a gamma efferent length reference signal. When this signal contracts the end muscles, the annulospiral sensor is stretched. Tracing the resulting signal to the motor neuron we find it has a positive effect, causing the main muscle to shorten and thus bringing the signal from the annulospiral sensor back toward zero: negative feedback. The muscle spindle is a neuro-mechanical comparator. The gamma efferent and alpha efferent signals making up the total reference signal for the inner loop are normally co-activated, although perhaps not in the way assumed in this model.

inverse dynamical equations. Instead, the entire system made of the neuromotor control systems and the mass of the arm constitutes an analog computer that directly solves the equations, automatically providing the driving signal E to the muscle that will produce the movements that continuously satisfy the equation. The "inverse" effects are implicit in the negative feedback loops. With the various K-factors set for reasonable loop gain and critical damping, the angle of the arm will follow rapid changes in the reference signals, so any desired trajectories of movement can be created simply by patterning the changes in reference signals.

2.4. A COMMENT ON THE HISTORY OF THINKING ABOUT CONTROL

This bootstrap way of solving differential equations by using "implicit solutions" and feedback was well known in the heyday of analog computing. When the digital computer took over, there was a transition to top-down or command-driven models, and the power of the digital computer removed any barriers to the use of complex matrix

calculations. The ease with which such calculations could be done made inverse-dynamic and -kinematic calculations seem perfectly reasonable; their enormous underlying complexity was never visible to the writer of programs in high-level languages, where a matrix multiplication could be commanded just by writing $M1\,M2$. The high precision of floating-point calculations concealed the problem of drift in integrations, which in devices of lower precision would have made open-loop inverse calculations grossly inaccurate after very short run-times.

So the digital computer, by making a top-down model seem feasible, replaced the old analog approaches in the minds of a new generation of modelers and engineers, and much of the old lore was lost, including the method of implicit solutions. This, I suggest, is why in modern models of arm control we find such extremely complex proposals, where a very simple and anatomically justified model will serve at least as well.

2.5. THE CONTROL MODEL VS. THE MASS-SPRING MODEL

In recent years, motor-control modelers have proposed a mass-spring model of limb control, in which a central program generator emits command signals which change the resting length of muscles and thus determine joint angles. The muscle model is essentially the same one used here. However, as we can see, there is no difference in form between the behavior of an open-loop mass-spring model and that of the control-system model so far developed on physiological grounds. Both behave in a way that can be described by a second-order differential equation.

The main difference is that the control-system analysis shows that the parameters of the mass-spring effect are not those of the muscle. The actual muscle spring constant appears only in the term that expresses the equivalent moment of inertia, where U appears. The apparent spring constant is determined completely by the feedback coefficients (if muscle viscosity had been explicitly represented, that term would appear in the velocity term of the equation). It should be relatively easy, therefore, to test whether the open-loop or the control model should be accepted. If the measured apparent spring constant is the same as that measured in muscle preparations, then the open-loop model is correct. That, however, would be most puzzling, because then there would be no function left for the known feedback loops.

The control model as developed so far, with stretch and tendon reflexes alone, behaves like a person with an injury that cuts off brainstem control. If the reference signals R_g and R_a are set to hold the arm out straight and level and gravity is then switched on, the arm will slowly sag as much as $45°$, a condition known as "waxy flexibility". This occurs when the parameters are adjusted so that repetitive square-wave changes in the reference signal are followed accurately by the arm with a time constant of about $0.1\,s$. The mass-spring model cannot account for this combination of fast responses to abrupt changes in the command signal, accompanied by large compliance to gravitational force. It is not very likely that the mass-spring model is correct even though it is based on a correct model of muscle action.

This concludes the basic description of the lowest level of control systems in the arm model. There is one such composite stretch–tension system for each of the three degrees of freedom of the arm.

2.6. THE SECOND KINESTHETIC LEVEL OF CONTROL

The control system so far consists entirely of spinal reflexes, with position being controlled only through sensing muscle length. Muscle length is not a very accurate measure of joint angle, partly because of the nonlinear relationship (which has been ignored above), and partly because the mechanical advantage is so poor (at the elbow, it is somewhere around 1:8). Furthermore, as we have seen, the parameters which optimize the reflex loops for fast control leave position control against sustained disturbances rather weak. The effective return spring constant is only $K_g/K_t + 1/(K_oK_t)$. A reasonable number of K_oK_t is 200, and K_g/K_o is approximately 3, leaving an effective spring constant about 3 n M per radian of deflection. This is why the spinal arm sags under gravity.

To improve position control, we introduce a second level of control based on direct sensing of the joint angle (Figure 4). Sensory signals affected by joint angle are known to rise through the spinal cord to about the brain-stem, and the reference signals for the spinal control loops originate no lower than that level. In the brain-stem, collaterals from the sensory signals cross over to the motor side, where they join, in motor nuclei, with "command" signals from still higher in the brain. An excitatory perceptual signal joins with an inhibitory "command" signal (coming in part from the cerebellum), providing a comparator function. There is some evidence that neural time-integration occurs in the downgoing path, probably in the brain-stem motor nuclei. The resulting output signal becomes the reference signal, for the spinal control loop.

Whether or not that evidence still holds up (it is backed up only by the author's rather aged memories), we supply an equivalent control system in the model because it is needed to produce accurate and disturbance-resistant position control.

The second level of control shown in Figure 4 is very simple. In each second-level system, the joint-angle signal enters a comparator along with a reference signal from higher systems. The difference between these signals, the second-level error signal, enters a neural integral-plus-proportional amplifier and the output signal sets both of the first-level reference signals, R_g and R_a (which are known to be coactivated under at least some circumstances). It is possible that the gamma signal should be co-activated with the reference signal coming into the second level, since position control is involved in both systems. This model is tolerant enough that several arrangements would work equally well. More detailed neuroanatomical information than is available would be needed to make this choice.

With this second level of kinesthetic control included, the sensed angular position of the joint closely follows the second-level reference signal for reasonable speeds of change. With optimal adjustments of parameters, square-wave changes in that reference signal are followed without overshoot by the joint angle signal with a time constant of about 0.15 s. Turning gravity on and off now has no perceptible effect on steady-state arm position.

The physical model of the arm converts applied torques into arm accelerations, velocities and positions results in numerous interactions among the degrees of freedom, including the effects of Coriolis forces during rapid movements. In general, the operation of one control system disturbs the operations of the others. The control systems quite automatically cancel most of the effects of these interactions. The design of the spinal
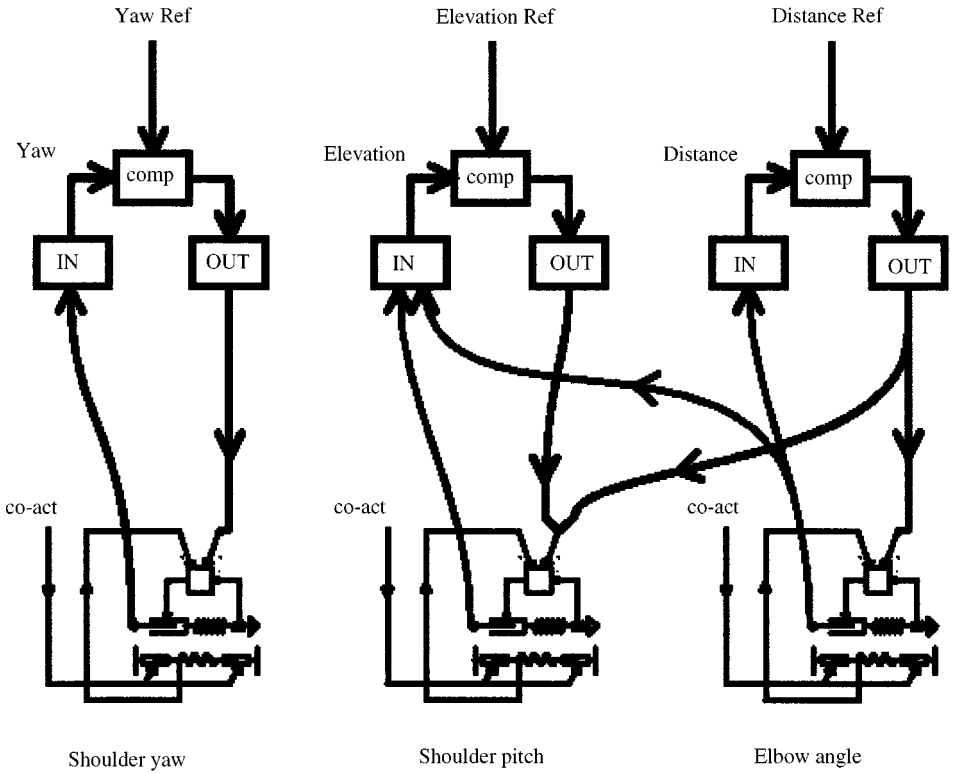
FIGURE 4. The two levels of kinesthetic control systems. See text.

reflexes, taken literally, is evidently sufficient to create independent control of each degree of freedom. It is comforting to a seat-of-the-pants engineer to know that a species could evolve such an elegant arrangement for itself by trial and error.

There is only one concession to compensating for these interactions, described below, and it is mainly a geometrical compensation, not a dynamic one.

2.6.1. Changing the coordinate system

If the position of a "fingertip" is calculated in terms of joint angles, the equations become complex, and an ambiguity develops as the elbow joint passes a 90° angle. For example, with the upper arm horizontal the vertical distance of the fingertip above an objective reference plane increases at first as the elbow bends, but then decreases again as it bends past the vertical. If the upper arm angle is changed the elbow angle where this ambiguity occurs changes.

The joint-centered coordinate system can easily be changed to an unambiguous shoulder-centered system in azimuth, elevation and radius. To convert to horizontal, vertical and radial fingertip control, two small details added to the second level of control suffice; refer to Figure 4.

First, the input function of the "elevation" control system receives both the signal representing the vertical joint angle at the shoulder and the signal representing elbow angle, giving an input weight of 1 to the shoulder angle and 0.5 to the elbow angle. The result is a sensory signal that is nearly proportional to the elevation angle of a line from the shoulder through the fingertip. The output of this system changes the first-level reference signal for shoulder vertical angle alone, which suffices to control the vertical angle of this line.

Second, the level-two system which senses elbow angle alone still senses elbow angle, but the error signal, amplified, is sent to both the first-level shoulder elevation and elbow-angle reference inputs, in the ratio of $1:2$. Thus changing the second-level reference signal for elbow angle causes the same change in elbow angle as before, but also a change in the vertical angle at the shoulder by half as much. The result is that as the elbow straightens, the shoulder angle increases (rises) by about half the amount, and the hand on the end of the arm moves nearly along a line from shoulder to hand. This greatly reduces the interaction between a change in elbow angle and a change in elevation of the line from shoulder to fingertip.

While the first level of control works in two coordinate systems, one centered at the shoulder and another at the elbow, the second level of control works in a single coordinate system centered on the shoulder. This small change converts the three complex degrees of freedom of the arm to a single polar coordinate system in which there are no ambiguities within the natural range of movement of the human arm. Higher systems which employ these two levels of kinesthetic control now have available three places to set reference signals: one for sideward position of the hand, one for vertical position and one for reach distance.

2.7. VISUAL CONTROL OF POINTING

The model presented here is actually part of a more extensive, but incomplete, model in which visual perception of the three-dimensional relationship between the fingertip and a moveable target is controlled by adjusting reference signals for the three two-level systems developed so far. The resulting three-level model does exhibit competent pointing behavior at stationary or arbitrary moving targets in three dimensions, in real time. However, problems remain in converting from the eye-centered binocular visual control systems to the shoulder-centered kinesthetic control systems. For sudden target jumps, the disparity between the coordinate system origins can result in initial fingertip movements in the wrong direction by $45°$ or more, which leads to unnatural trajectories. Slow target movements are accurately tracked; the basic problem exists only for maximum-speed movements.

Some hints as to the solution can be seen in the fact that if the second-level reference signals are suddenly changed (the visual system being inactive), the trajectories of fingertip movement are quantitatively similar to those seen in natural sudden arm-movements. Some kind of mapping compensation is necessary for the visual target positions to become the correct kinesthetic target positions—either on the input or the output side of the visual control systems.

Because of these remaining problems, the rest of the discussion is confined to the kinesthetic levels of control.

2.8. SOME DESIGN DETAILS

The inner loop of the first level of kinesthetic control, the tendon reflex, seems to require a relatively high gain to operate realistically. Stabilization of this loop, which does not involve the inertia of the arm as a whole, requires a high-frequency rolloff which is achieved by slowing the muscle-tension response to the first-level error signals (the motoneuron outputs). This corresponds to the effect of a parallel damping element in the muscle model. The resulting muscle-tension response to a step-change in driving signal is a negatively accelerated exponential rise to an asymptote. This response represents the introduction of muscle viscosity, although in a somewhat informal way. In a more complete model, the actual tension–velocity curves would be used. The closed-loop rise time for altering muscle tension can in principle be as short as twice the propagation delay in this loop, or about 10 ms.

The neural time delays in the real system are approximated in the model by the choice of dt, the real-time equivalent of one iteration of the model calculations. The spinal loops are calculated four times for each iteration of the higher levels of control, so that the effective delay in the second kinesthetic level is four times the delay in the first (two-loop) level. Other than this provision, no serious attempt was made to match the model's delay exactly to those measurable in the real neuromuscular system.

The integrations in the physical model of the arm are simple summations (Euler integrations) with no attempt to made them more accurate. Because of the closed-loop nature of this model, integration errors do not accumulate; they act more like small disturbances of the control systems, which are removed by the control action. Tests (done by Gregory Williams) comparing more exact integration methods with the simple summation showed no discernible differences in behavior of the model.

Finally, the calculation of the rate-of-change component of the spindle response is done by taking first differences of the length-error signal and passing the result through a one-stage smoothing filter with a short time constant. This serves to approximate the rounded-off first-derivative response seen in the real system (McMahon, 1983, Figure 6.8, p. 153).

The multiplicity of calculations leaves room for conceptual error; the model as presented should be used only for general evaluation, not as a source of parameter values for the real arm-control system (without independent confirmation of the calculations).

3. Performance of the model

Figure 5 is a snapshot of the screen presented to the user, showing plots of variables on the left-hand side, a fused binocular view of the fingertip (a dot) and target (an upside-down triangle) in the center, and a rotatable perspective three-dimensional view of a "Little Man" on the right. The eyes individually track the target using elementary feedback control loops in x and y, with head movement being a fraction of average eye movement. Depth information is obtained from image disparity. The visual control systems are included in the distribution source and executable code, but we discuss here the performance of the model in the test modes which involve only kinesthetic control.
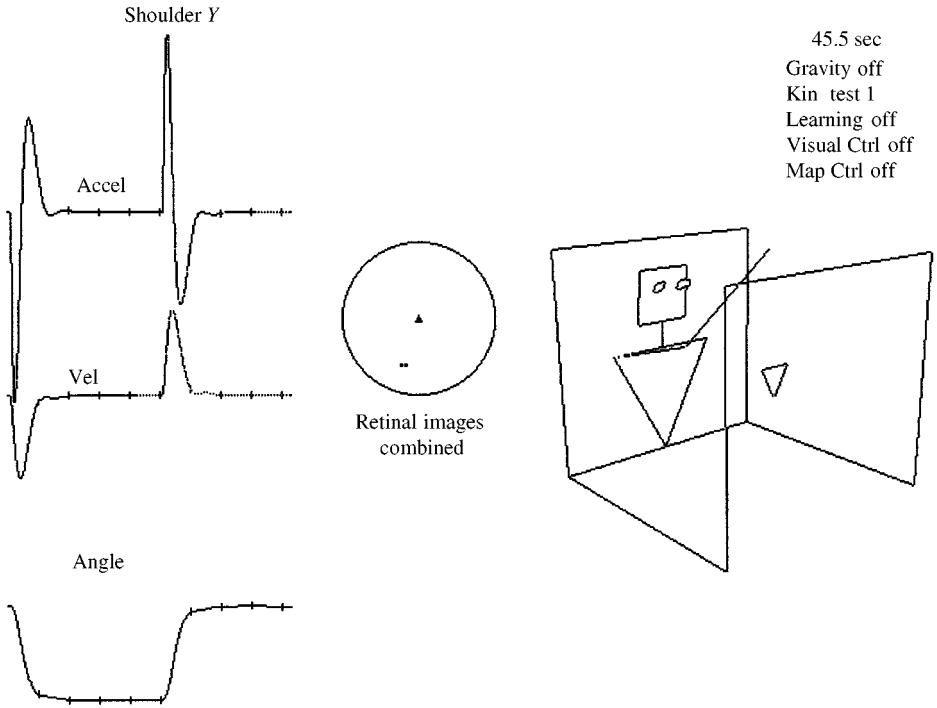
FIGURE 5. Performance of the Model. Test Mode 2, showing the result of applying a square-wave reference signal to the shoulder pitch control system of the first level. Gravity is off. The circle in the center is the combined left and right visual fields. The eyes are converged on the target which shows as a single (upside-down) triangle. The fingertip, which is at a different distance from the eyes, shows up as a pair of dots below the target (because, as seen on the right, the fingertip is above the target). Visual control is not working in this test mode. The time ticks on the plot represent tenths of a second in real time.

3.1. TEST MODES 1, 2 AND 3: STRETCH AND TENDON REFLEX CONTROL

In these modes, the reference signals for the three first-level kinesthetic control systems are square waves for the system under test and constant for the other two systems. As these are approximate joint-angle reference signals, perfect control would entail the joint angle (bottom trace) executing a perfect square wave. The actual response seen in Figure 4 is for the elevation control system with the arm at slightly less than full extension, and shows the behavior of the angle from shoulder to fingertip. The time constant of the response is about 0.1 s. Gravity is switched off for this plot.

There are five parameters associated with each control system, adjustable from the keyboard. They are the tension perceptual sensitivity K_t, the proportional length-sensing sensitivity K_g, the length-rate sensitivity (damping coefficient) K_d, the muscle sensitivity in terms of contraction per unit error signal K_o, and the series muscle spring constant K_s. K_1 is fixed at 1. The performance shown in Figure 4 for the elevation control system shows the result of optimal (trial-and-error) adjustment of these parameters.
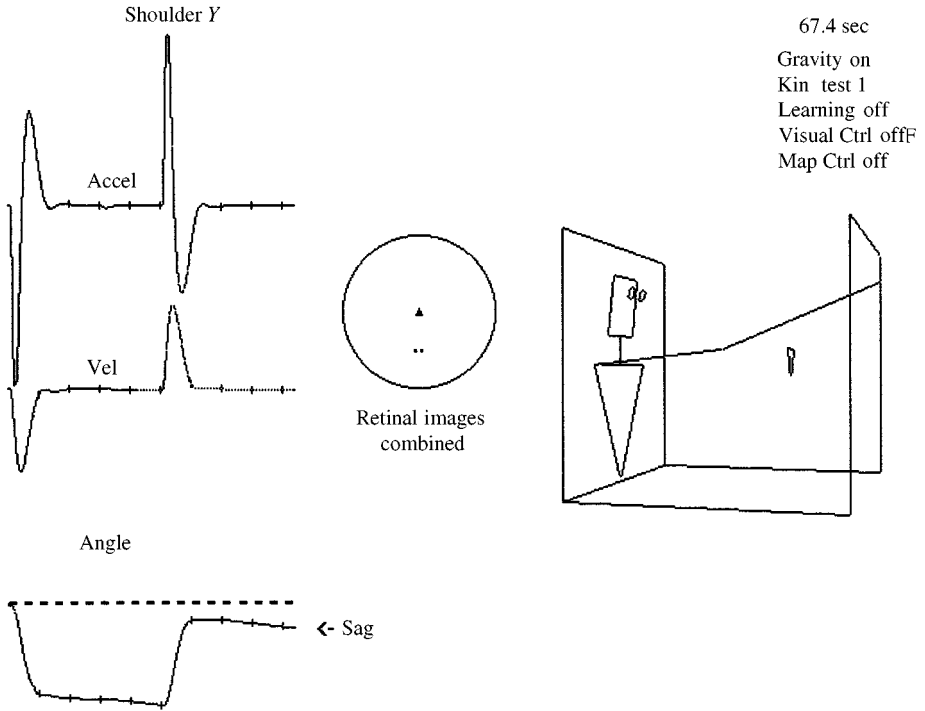
FIGURE 6. Test Mode 2, with gravity switched on. Note that speed of response remains high, but gravity causes a gradual sag in arm position.

The other two control systems show essentially the same behavior with proper adjustment of their parameters. Their time constants are also about 0.1 s. There are almost no residual effects from dynamical and kinematic interactions among the arm segments. Note that the behavior of the controlled angles is well behaved, with minimal overshoot or oscillation. The muscle tension is proportional to the acceleration plot, top left.

The conditions for Figure 6 are the same as for Figure 5, except that at the start of the plot, gravity is switched on. The square-wave pattern immediately begins to sag downward. This is the "waxy flexibility" effect noted above.

3.2. TEST MODES 4, 5, AND 6: SECOND LEVEL JOINT-ANGLE CONTROL

Figure 7 shows the behavior of the elevation angle with both levels of kinesthetic control active and a square-wave reference signal being sent to the second level reference input. The response is a little slower. Now, however, switching gravity on and off makes no perceptible difference in the plots.

4. Conclusions

This model is clearly a good deal simpler than other models that require the nervous system to perform complex inverse calculations and to specify trajectories as analytical
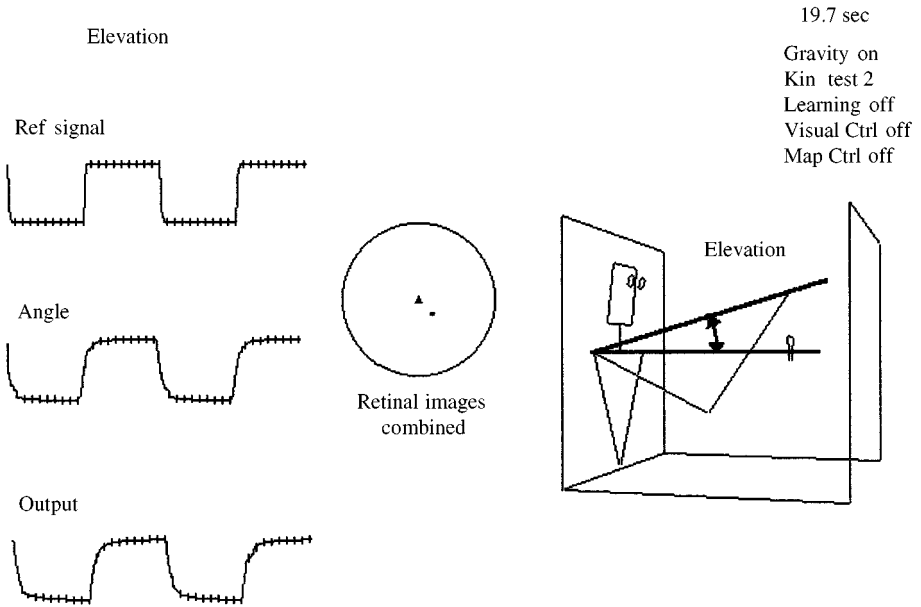
FIGURE 7. Test Mode 5, elevation control. The reference signal now defines the angle from shoulder to fingertip relative to the horizontal, independently of the elbow angle. Time scale is now slower as can be seen from the 0.1 s ticks. The top plot is the reference signal square wave: the next plot is the actual angle that results. Bottom plot is the output signal going to the first-level control systems. Note that in this very simple model the output does not contain any first-derivative component to speed the response.

curves. The analog computations that do take place are specified by a handful of anatomically meaningful parameters, without requiring the control system to know the physical properties of the world on which it acts (such as moments of inertia). Specifications for the desired results of action are given in the form of simple one-dimensional reference signals against which one-dimensional perceptual signals are compared. Any desired trajectories of movement can be created simply by providing reference signals that vary as the associated perceptual signals (and the physical variables on which they depend) are to vary. I hope that the simplicity of this model will suggest a reconsideration of some of the old lore of analog computing, and even of control theory in the days before it became almost unrecognizeably complex. Much of the complexity of today's theories may arise simply from having forgotten the lessons of the past.

References

HUTCHINSON, S., HAGER, G. D. & CORKE, P. I. (1996). A tutorial on visual servo control. *IEEE Transactions on Robotics and Automation*, **12,** 651–670.
RIVIN, E. E. (1988). *Mechanical Design of Robots.* New York: McGraw-Hill.
MCCLOY, D. & HARRIS, D. M. J. (1988). *Robotics: An Introduction.* New York: Halsey Press.
MCMAHON, T. A. (1984). *Muscles, Reflexes and Locomotion.* Princeton: Princeton University Press.

Related reading

BAYLIS, L. E. (1966). *Living Control Systems*. San Francisco: W. H. Freeman.

CRAIK, K. J. W. (1947) Theory of the human operator in control systems. I. The operator as an engineering system. *British Journal of Psychology* (*General Section*) **38**, 56–61.

ELLSON, D. G. (1949) The application of operational analysis to human motor behavior. *Psychological Reviews*, **56,** 9–17.

KORN, G. A. & KORN, T. M. (1952). *Electronic Analog Computers*. New York: McGraw-Hill.

MCFARLAND, D. J. (1971). *Feedback Mechanisms in Animal Behavior*. London and New York: Academic Press.

SOROKA, W. W. (1954). *Analog Methods in Computation and Stimulation*. New York: McGraw-Hill.

TAYLOR, F. V. & Birmingham, H. P. (1948). Studies of tracking behavior. I. The acceleration pattern of quick manual corrective responses. *Journal of Experimental Psychology*, **38,** 783–795.

TOATES (1975). *Control Theory in Biology and Experimental Psychology*. London: Hutchison Educational.

WIENER, N. (1948). *Cybernetics*. New York: Wiley.